# Framework for Engineering the Collective Behavior of Complex Rhythmic Systems

**Craig G. Rusin,[†] István Z. Kiss,[‡] Hiroshi Kori,[§,⊥] and John L. Hudson*[,†]**

*Department of Chemical Engineering, University of Virginia, Charlottesville, Virginia 22904, Department of Chemistry, 3501 Laclede Ave, Saint Louis University, St. Louis, Missouri 63103, Division of Advanced Sciences, Ochadai Academic Production, Ochanomizu University, Tokyo, 112-8610, Japan, and PRESTO, Japan Science and Technology Agency, Kawaguchi 332-0012, Japan*

We have developed an engineering framework which utilizes experiment-based phase models to tune complex dynamic structures to desired states; weak, nondestructive signals are employed to alter interactions among nonlinear rhythmic elements. In this manuscript, we present an integrated overview and discussion of our recent studies in this area. Experiments on electrochemical reactions were conducted using multielectrode arrays to demonstrate the use of mild model-engineered feedback to achieve a desirable system response. Application is made to the tuning of phase difference between two oscillators, generation of sequentially visited dynamic cluster patterns, engineering dynamically differentiated cluster states, and to the design of a nonlinear antipacemaker for the destruction of synchronization of a population of interacting oscillators.



## I. Introduction

Organized dynamical behavior spontaneously emerges in many complex chemical and biological systems due to interactions between discrete subunits. Examples of such behavior include coherent light emissions from lasers,[1] propagating electrochemical waves in cardiac systems, and synchronization of biological neurons.[2] The properties of these large-scale emergent behaviors depend on the behavior of the constituent parts as well as the type and extent of their interactions.[3,4] The efficient description and design of a complex dynamic structure is a formidable task that requires simple yet accurate models incorporating integrative experimental and mathematical approaches that can handle hierarchical complexities and predict emergent, system-level properties. Such approaches include phase models[5−7] and pulse-coupled models[8] which have been used to describe mutual entrainment of weakly interacting neuronal assemblies.[9−11]

A major question of both theoretical and practical importance is how to bring the collective behavior of a rhythmic system to a desired condition or, equivalently, how to avoid a deleterious condition without destroying the inherent behavior of its constituent parts.[12] For example, neurological diseases such as epilepsy and essential tremors are characterized by pathological synchronization of a group of neurons within the brain of a patient.[13] Since the neural oscillators are synchronized, the mean signal produced by the collective behavior of the population (EEG) is also oscillatory. In these medical applications, it is desirable to disrupt the overall oscillatory signal, terminating its associated physiological effect.[14] Collective oscillations can be eliminated by either stopping the rhythmic activity of the individual oscillators or by desynchronizing the population of oscillators such that the individual oscillators become disorganized, causing a steady mean (overall) signal. While the former typically requires a sizable input such as the introduction of large, rapid voltage pulses (such as those used in current deep brain stimulation therapies[15]), desynchronization can be effected with the use of mild input signals which have minimal effect on the individual elements in the population. Thus, the objective in this case is to determine an effective yet mild external signal which will steer a system toward the desired dynamic state.

This contribution provides an integrated overview and discussion of our recent studies[16−18] on the development of a general methodology for controlling the collective behavior of a population of rhythmic units. We use as an experimental model system the electrodissolution of nickel on electrode arrays, which are held under conditions (electrolyte concentration and applied potential) such that the rate of dissolution is oscillatory in time. We demonstrate that nonlinear feedback loops can be rigorously designed using experiment-based phase models[3,5−7,19] to “dial up” a desired collective behavior without requiring detailed knowledge of the underlying physiochemical properties of the target system. Weak feedback signals are designed so as to have a minimal impact on the dynamics of the individual electrodes while producing a collective behavior of the population that is both qualitatively and quantitatively different than the dynamic behavior of an uncontrolled system.

We demonstrate the utility of the methodology with the systematic design of a variety of dynamical behaviors: phase locking with a preset phase difference with two oscillators, sequentially visited dynamic cluster patterns with a small set of four oscillators, and various types of stable clusters with a population of sixty-four oscillators. We also show the power of noninvasive, model-engineered feedback in effectively achieving desynchronization and order reduction in populations of synchronized oscillators; the experiment-based phase model is an essential component in obtaining the optimal nonlinear antipacemaker design.

## II. Methodology

Our framework is based on phase models, in which the state of the individual rhythmic elements of a system is approximated by their position along their respective limit cycle trajectories.[5,6] Under this approximation, a population of oscillators with weak heterogeneities and weak, global (all-to-all) interactions can be described by[5]

* To whom correspondence should be addressed. E-mail: hudson@virginia.edu.
[†] University of Virginia.
[‡] Saint Louis University.
[§] Ochanomizu University.
[⊥] PRESTO.

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$$\frac{\mathrm{d}\phi_i}{\mathrm{d}t} = \omega_i + \frac{K}{N}\sum_{j=1}^{N} H(\phi_j - \phi_i) \tag{1}$$

where $\phi_i$ and $\omega_i$ are the phase and the natural frequency of the $i$th oscillator, $K$ is the global coupling strength, and $H$ is the interaction function. Equation 1 shows that the phase of an element increases at a rate equal to its inherent frequency ($\omega_i$), slightly modified by slowing down or speeding up due to interactions with other elements. The interaction function $H(\Delta\phi)$ characterizes the extent of phase advance or delay as a result of interaction between oscillators. For a desired target state (i.e., time variation of the phases of the oscillators), an optimal target interaction function $H(\Delta\phi)$ is determined through analytical and numerical investigations of the phase model (eq 1). The manipulation of the harmonics of the interaction function provides flexibility in developing a stimulation which produces a desired state. While this methodology is demonstrated here with global coupling, other coupling topologies can also be used.

**Description of Method.** The general methodology for producing a desired collective behavior[16−18] within a target system follows the scheme shown in Figure 1. The flexibility in obtaining phase model interaction functions can be exploited to derive specific feedback schemes capable of obtaining desired dynamic complex states. This is done as follows:

**0. Pick a Desired Dynamic State for a Set or Population of Rhythmic Elements.** For example, say that we are dealing with a synchronized population of oscillators. One important goal could be to disrupt the synchronization leading to a desynchronized system. Or, in other cases starting from a disordered state, the desired dynamic state could be a synchronized population or a population with stable or intermittent clustering. Each of these examples will be considered below.

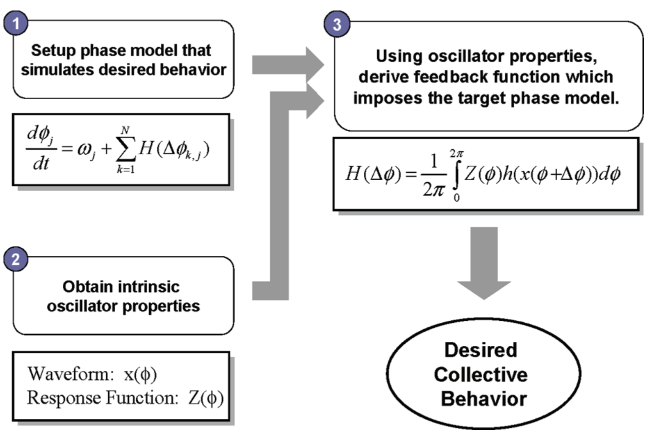


**Figure 1.** General methodology for designing complex dynamical structures using phase model description.

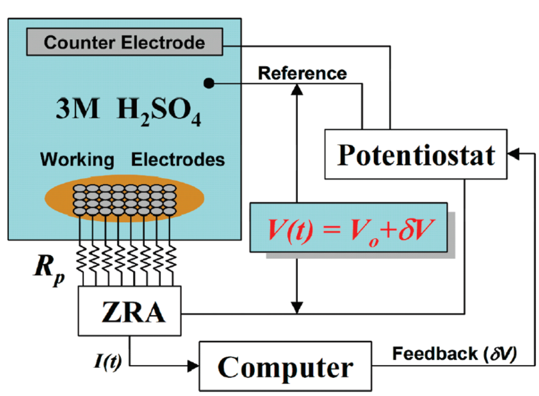


**Figure 2.** Schematic diagram of the experimental apparatus. $R_p$ represents a set of 650 Ω resistors. The computer is a real time data acquisition computer, which calculates the feedback signal, $\delta V$.

**1. Find a Phase Model That Reproduces the Target Dynamics.** Once a target collective behavior has been identified, the shape of the corresponding interaction function must be determined; this is an active area mathematical research.[5,12,20−23] For example, the existence of balanced cluster states can be determined by examining the transversal eigenvalues of these states as derived by Okuda.[24] As demonstrated below, we find a target interaction function and a coupling strength and topology that will produce the desired dynamic behavior.

**2. Determine the Canonical Properties of the Oscillations (Waveform, Frequency, Response Function).** As it is shown below, the procedure requires the measurement of the response function and the waveform of the oscillators. The waveform of oscillation can usually be directly measured with standard data acquisition equipment. The response function $Z(\phi)$, proportional to the phase response curve (defined for weak stimuli) commonly used in circadian rhythms to interpret external entrainment, shows the phase advance per unit perturbation as a function of the phase of the oscillator. The response function can be obtained directly from experiments by perturbing a single oscillator with weak pulses and measuring the associated phase response.[7,9,25] The response function may also be obtained indirectly by measuring the interaction function between two oscillators. In this case, the period of two weakly interacting oscillators is measured as a function of their phase difference, $P(\Delta\phi)$; the interaction function can then be determined

$$H(\Delta\phi) = \frac{-2\pi}{K}\left(\frac{P(\Delta\phi) - P_{\mathrm{base}}}{P_{\mathrm{base}}^{\ 2}}\right) \tag{2}$$

where $P_{\mathrm{base}} = 1/\omega$.[26] The response function can then be calculated from the relation:[5]

$$H(\Delta\phi) = \frac{1}{2\pi}\int_0^{2\pi} Z(\phi)h(\phi + \Delta\phi)\,\mathrm{d}\phi \tag{3}$$

where $h(\phi)$ is the coupling function, representing the physical interaction between the elements. Equation 3 can be analytically solved for $Z(\phi)$ in the Fourier space, yielding the linear system

$$\begin{bmatrix} C_n & D_n \\ D_n & -C_n \end{bmatrix}\begin{bmatrix} A_n \\ B_n \end{bmatrix} = 2\begin{bmatrix} R_n \\ S_n \end{bmatrix} \tag{4}$$

where $A_n$ and $B_n$ are the even and odd Fourier coefficients of $Z(\phi)$, respectively, $C_n$ and $D_n$ are the even and odd Fourier coefficients of $h(\phi)$, respectively, and $R_n$ and $S_n$ are the even and odd Fourier coefficients of $H(\Delta\phi)$, respectively. This method was utilized to determine the response function of the electrochemical oscillators used in this work.

**3. Design a Feedback Stimulation to Reproduce the Target Interaction Function.** We engineer the desired behavior of a population of $N$ oscillators through the imposition of nonlinear, time-delayed feedback. A time-dependent system parameter perturbation, $\delta p(t)$, is chosen to be a nonlinear function of the measured variables $x_k(t)$ summed over the population

$$\delta p(t) = \frac{K}{N}\sum_{k=1}^{N} h(x_k(t)) \tag{5}$$

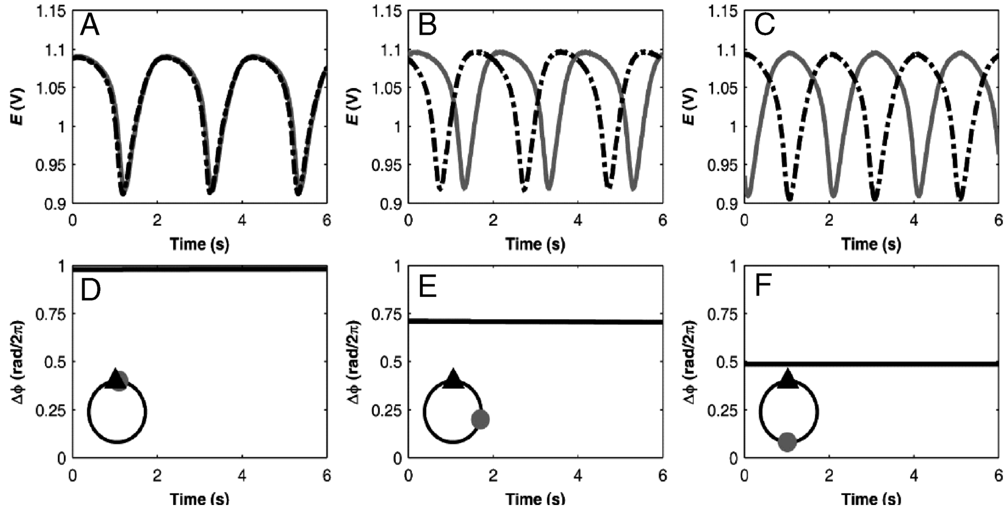


**Figure 3.** Tuning the phase difference between two oscillators with nonlinear feedback:[18] time series of the electrode potential (A−C) and phase difference (D−F) of a system of two elements with second order global feedback, [$K = 0.03$, $k_0 = 0.03$ V, $k_1 = 1.72$, $k_2 = -4.6816$ V$^{-1}$, $\tau_1 = 0.012$ rad/2$\pi$, $\tau_2 = 0.143$ rad/2$\pi$]. (left column) In-phase synchronization ($\Delta\tau = 0$ rad/2$\pi$). (middle column) Out-of-phase synchronization with phase difference of $\pi/2$ ($\Delta\tau = 0.23$ rad/2$\pi$). (right column) Antiphase synchronization ($\Delta\tau = 0.5$ rad/2$\pi$). The phase loop diagrams indicate the relative position of the elements in the system and the direction of rotation.

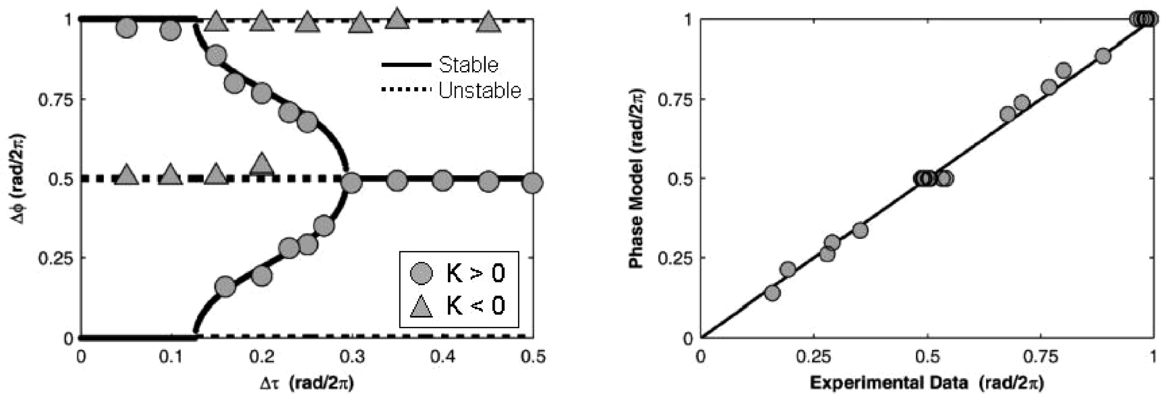


**Figure 4.** (A) Stationary phase difference values of a system of two rhythmic elements under second-order feedback [$K = 0.03$, $k_0 = 0.03$ V, $k_1 = 1.72$, $k_2 = -4.6816$ V$^{-1}$, $\tau_1 = 0.012$ rad/2$\pi$, $\tau_2 = 0.143$ rad/2$\pi$].[18] Lines represent phase model predictions of the stable (solid) and unstable (dotted) stationary states. Experimental measurements using positive feedback (circles) and negative feedback (triangles) are superimposed. (B) Parity plot of phase model predictions versus experimental measurements.

$K$ is the overall gain; here, we choose the nonlinear feedback to be a polynomial:

$$h(x) = \sum_{n=0}^{s} k_n x(t - \tau_n)^n \tag{6}$$

where $k_n$ and $\tau_n$ are the gain and the delay of the $n$th order feedback, respectively; $S$ is the overall order of the feedback.

The challenge is obtaining the best form of the feedback, that is, obtaining the order and time delays best suited for the desired complex structure through the phase model eq 1. The feedback parameters for use in the experiments (eqs 5 and 6) can be obtained from eq 3. Given a feedback $\delta p(t)$ and a response function $Z(\phi)$ we could obtain the interaction function $\mathrm{H}(\Delta\phi)$ for use in the phase model. However, we proceed in the opposite manner and choose a target interaction function which produces desired states, and then design a feedback loop $\delta p(t)$ with optimized feedback gains $k_n$ and delays $\tau_n$ to give the desired $\mathrm{H}(\Delta\phi)$. The parameters $k_n$ and $\tau_n$ are found with standard optimization techniques.[16,17] It can be shown analytically that in weakly nonlinear oscillators the order of feedback enhances the corresponding harmonic in the interaction function and the delay time produces an offset in the phase difference.[16,17] Thus, if we need an interaction function with predominantly first and second order harmonics, linear and quadratic feedback shall be applied and the delays of the feedback used to tune the ratio of the cosine and sine terms of $H$. The optimized feedback is expected then to produce the target dynamics through imposing the proper interaction function in the phase model description.

**4. Apply the Feedback to Populations.** A real-time data acquisition and control system can be used to implement the nonlinear, time-delayed feedback procedure developed in step 3.

**Limitations of Method.** There are limits on how well this methodology can be applied to rhythmic systems. It is well-known that the phase approximation does not account for amplitude effects[5] and the method eventually breaks down as the interaction strength increases. Feedback stimulation must be weak so as to not disturb the shape of the waveform of the oscillator. Moreover, when the time-delay is not small (compared to $K^{-1}$ in eq 1), the effect of delay cannot be approximated by a phase offset in the interaction function[27] and the feedback parameters cannot be accurately determined. The method can be applied to both smooth and relaxational oscillators. The experiments described below are done with a mildly heteroge-

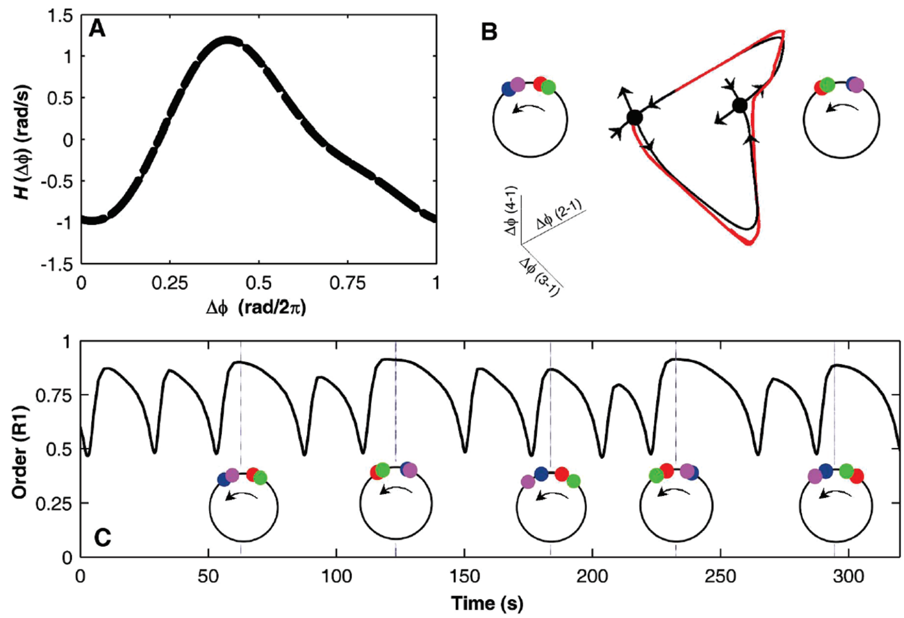


**Figure 5.** Engineering a system of four nonidentical oscillators to generate sequential cluster patterns.[16] (A) Target [solid line, $H(\Delta\phi) = \sin(\Delta\phi - 1.32) - 0.25\sin(2\Delta\phi)$] and optimized (dashed line) interaction function with feedback parameters $k_0 = -0.0526$ V, $k_1 = 8.7376$, $k_2 = 16.3696$ V$^{-1}$, $\tau_1 = 0.21$, $\tau_2 = 0.68$, $K = 0.0494$ ($V = 1.165$ V, $R_{tot} = 162.5$ Ω). (B) Theoretical (black line) and experimentally observed (red line) heteroclinic orbits and its associated unstable cluster states. (C) Time series of the order parameter [$Rk = 1/N|\sum_{j=1}^{N} \exp(ik\phi_j)|$, $k = 1$], and associated cluster configurations.

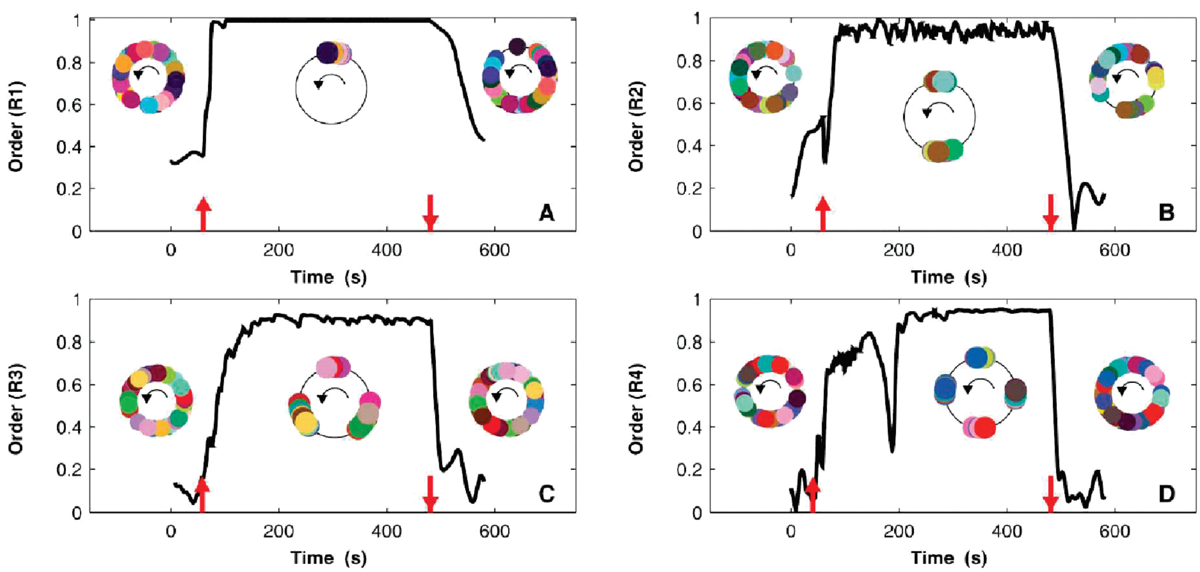


**Figure 6.** Engineering cluster patterns.[17] (A) Time series of the R1 order parameter, using feedback optimized to produce a one cluster state. Arrows indicate the application and termination of the feedback signal. (B) Time series of the R2 order parameter using feedback optimized to produce a two cluster state and time series of the R3 order parameter using feedback optimized to produce a three cluster state. (D) Time series of the R4 order parameter using feedback optimized to produce a four cluster state.

neous population and in the presence of unavoidable noise; nevertheless the method was successfully used in engineering desired collective behavior. However, it is unknown where the method will break down in the presence of severe noise and levels of heterogeneities (such as chaotic systems or systems composed of heterogeneous oscillator types). General methods for obtaining the phase model description are not available for experimental systems consisting of large populations of coupled oscillators. In order to apply nonlinear feedback, the time series from each oscillator must be measured; extensions to the method which require only a mean signal have been proposed for the case of linear feedback.[28] This discussion is limited to globally coupled systems; other factors come into play for more complex coupling topologies. Current research is underway on investigating and overcoming these limitations.

## III. Experimental Setup

A standard electrochemical cell consisting of an array of nickel working electrodes, a $Hg/Hg_2SO_4/K_2SO_4$ reference electrode, and a platinum mesh counter electrode was used. Experiments were carried out in 3.0 M $H_2SO_4$ solution at a temperature of 11 °C. (A constant low temperature reduces the dissolution rate and improves reproducibility.) A schematic of the apparatus is shown in Figure 2. The 64-electrode array in an 8 × 8 geometry is shown; for experiments with a single oscillators the array is replaced with one electrode. The working electrodes (1 mm diameter) are embedded in epoxy; the dissolution reaction takes place only at the exposed ends. The current produced by each electrode is independently measured

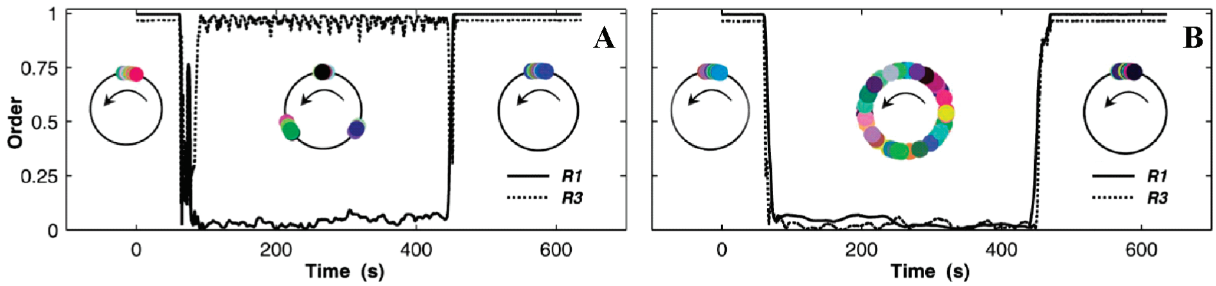


**Figure 7.** Desynchronization of a system of 64 coupled relaxation oscillators:[16] $V = 1.250$ V, $R_{tot} = 10.1\ \Omega$. (A) Time series of the order parameters R1 and R3 before, during (61 s < $t$ < 445 s), and after the application of linear time-delayed ($K = 1$, $k_1 = -1$, $\tau_1 = 0.016$) feedback to a coupled ($\varepsilon = 0.3$) population. (B) Time series of order parameters before, during (60 s < $t$ < 440 s), and after the application of nonlinear feedback ($k_0 = 1.41$ V, $k_1 = -1.09$, $k_2 = -5.35$ V$^{-1}$, $\tau_1 = 0.01$, $\tau_2 = 0.09$, $K = 0.55$).

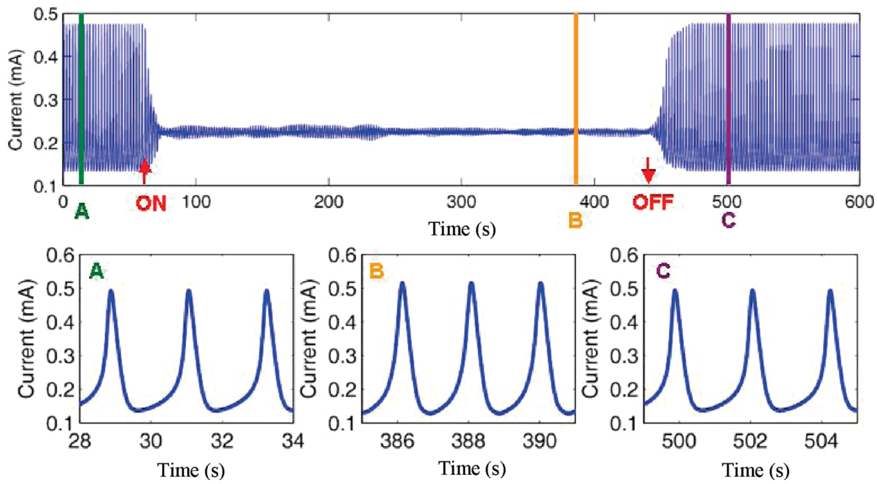


**Figure 8.** (top) Mean global signal of coupled relaxational oscillator population before, during, and after application of nonlinear feedback. Arrows indicate the application and removal of feedback. (bottom) Time trace of a representative individual oscillator at the indicated times (A, B, and C).

and sampled at 250 Hz, allowing the rate of reaction to be determined as a function of position and time.

In the feedback experiments, the measured observable is the current from which the electrode potential can be calculated; the control system parameter is the circuit potential ($V$). Additional details on the experiments can be found in the original papers.[16−18]

## IV. Results

**Tuning the Phase Difference between Two Oscillators.** We first demonstrate the method with a conceptually simple, yet nontrivial example: tuning the (phase-locked) phase difference between two electrochemical oscillators with different inherent frequencies. The electrode potential $E$(t) in the experimental chemical system is oscillatory (Figure 3A). The phase difference between two noninteracting electrodes with different frequencies increases linearly over time. We choose here, for illustration, three special target states of in-phase, out-of-phase, and antiphase entrainment which correspond to phase differences of $\Delta\phi^* = 0$, $\pi/2$, and $\pi$, respectively. Each of these states can be obtained with an interaction function consisting of first- and second-order harmonics of the form $H(\Delta\phi) = \sin(\Delta\phi - \Delta\tau) + {}^1/_2\sin(2\Delta\phi - 2\Delta\tau)$, with $\Delta\tau = 0$, $0.5\pi$, and $\pi$ rad, for in-phase, out-of-phase, and antiphase entrainments, respectively. (Additional details on the derivation of the target interaction function are given in ref 18.) Because the target interaction function is composed of first- and second-order harmonics, a feedback composed of linear and quadratic terms is chosen; the design requires that the waveform and the response function of the oscillators can be readily obtained.[16−18] Applying the designed feedback to the experimental system produced the desired phase locked states (Figures 3A−F). Furthermore, by experimentally measuring the locked phase difference as a function of $\Delta\tau$, we can see that it is possible to produce any phase difference between 0 and $2\pi$. The phase model accurately predicts all experimental observations (Figure 4A and B). Negative feedback was used to stabilize previously unstable states.

**Generation of Sequential Dynamical States.** We now consider the generation of sequential dynamical states.[29,30] The mathematical concept of slow switching[21,22] predicts an alternation between unstable synchronized cluster states in a population of (at least) four oscillators with, for example, $H(\Delta\phi) = \sin(\Delta\phi - 1.32) - 0.25\sin(2\Delta\phi)$ in eq 1. Because heteroclinic orbits connect the unstable dynamic states and these orbits are typically not robust against heterogeneities and noise caused by their structural instabilities, their demonstration in an experimental system is a challenging task.

We designed a quadratic feedback signal to produce an interaction function which has been proposed for slow switching (Figure 5A) in a population of four oscillators. The feedback signal caused the experimental system to sequentially visit multiple (unstable) two-cluster states, consisting of two oscillators in each cluster; Figure 5B shows two (saddle type) cluster states in state space. In the experiments, we observed switching between the cluster states (red line) along the theoretically predicted orbit (black line). We observed many switches along the heteroclinic orbits in a long time series. These switches can be seen as a fluctuation of the system order (Figure 5C). The time scale of the cluster switching is 60 s, much greater than the 2.2 s period of the individual oscillating elements.

**Engineering Dynamically Differentiated Clusters.** In each of the previous applications, there has been a single preselected target interaction function which corresponds to the desired dynamical behavior. However, many dynamical behaviors often have a family of associated interaction functions; it is often difficult to determine a priori which function is optimal. An example is the generation of phase clusters. It can be shown[17] that nearly balanced $M$-cluster solutions can be stable, if the following conditions are satisfied for the interaction function $H(\Delta\phi) = \sum A_k \sin(k\Delta\phi) + B_k \cos(k\Delta\phi)$:

(i) for any $k$, $B_k \approx 0$

(ii) for $k = M$, $A_k$ is a large positive number

(iii) for $k \neq M$, $A_k$ are small negative numbers

These general rules follow from the calculation of the transversal eigenvalues of balanced cluster states as derived by Okuda.[24] Mild feedback implies that the feedback gains, $k_n$, should be small. In this case, a multiobjective feedback optimization is constructed to select the target interaction function in which the strength of the feedback signal (e.g., $\sum k_n$) is minimized on the family of interaction functions that produces an $M$-cluster state. The result of the application of optimized feedbacks to a 64-oscillator system are shown in Figure 6. Predominantly linear feedback causes one cluster, quadratic feedback two-cluster, cubic feedback three-cluster, and quartic feedback four-cluster states.[17]

**Desynchronization.** Our proposed phase model methodology provides an efficient design of mild nonlinear feedback antipacemakers for weakly interacting systems. A system of 64 weakly relaxation oscillators, synchronized with global coupling through a common resistor[7] in a synchronized (one-cluster) state with large order parameter (left part of Figure 7A). Although the one-cluster state can be broken with linear feedback,[31] in some cases, synchronized cluster states may appear (here a three cluster as seen in Figure 7A) instead of a desynchronized state. The occurrence of spurious clusters is caused by the presence of higher harmonics within the net interaction function which is a superposition of the individual interaction functions due to coupling and linear feedback.

A desynchronized state without any stable cluster states can be obtained with nonlinear feedback. Mild, effective desynchronization can be achieved by minimizing the power of the feedback signal under the condition that the feedback produces a family of target interaction functions with negative odd components, e.g., $H = -\sin(\Delta\phi) - \sum_{k=2}^{M} \varepsilon_k \sin(k\Delta\phi)$. A linear programming optimization[16] resulted in a mild, second-order feedback that produces an interaction function with negative odd harmonics. This quadratic feedback successfully desynchronized the system as seen in Figure 7B. The initially synchronized state ($t < 60$ s) is desynchronized upon the application of the designed nonlinear feedback (60 s $< t <$ 440 s); the elements almost uniformly populate the cycle and all order parameters drop to low values. When the feedback is turned off ($t = 440$ s), the system returns to its original synchronized state. When actively desynchronizing the system using feedback, the global mean signal of the population was dramatically reduced, while the rhythmic behavior of the individual elements of the population remained undisturbed (Figure 8A and B).

## V. Concluding Remarks

We have reviewed our recent studies[16−18] on developing an effective method of designing complex dynamic structure and tuning a wide spectrum of emergent collective behavior in systems composed of rhythmic elements. The method is precise enough to engineer delicate synchronization features of nonlinear systems and can be applied to both small sets and large populations. Since the method does not require detailed a priori physical, chemical, and biological models, it may find applications in pacemaker and antipacemaker design in systems where there is a need for tuning complex dynamical rhythmic structures but where such detailed models are difficult to obtain.

## Acknowledgment

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